SPARROW DATASET ANALYSIS REPORT

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CONTENTS

1 INTRODUCTION

Reproductive success in bird populations is influenced by a variety of factors, including fitness, age, competition, nesting site quality, and environmental conditions. This study analyzes a 19-year observational dataset of song sparrows on a Pacific Northwest island to investigate these factors. These sparrows are monogamous, with females beginning to mate at one year old and often reusing the same nesting sites due to their territorial nature, even under suboptimal conditions. The dataset includes detailed information on individual sparrows, such as their age, nesting location, the year they were tagged, and the number of offspring produced. It also captures broader variables, such as the year of the study and population density, offering a comprehensive view of factors influencing reproductive success.

The primary objective of this analysis is to understand the ecological and biological sources of variability in reproductive success among song sparrows. Specifically, this study addresses the following questions:

- How do individual-level factors, such as age or nesting location, and population-level variables, such as competition (population density), influence reproductive success?
- Are there observable trends or patterns in reproductive success across different years or cohorts of sparrows?
- How do individual characteristics and environmental factors interact to influence variability in reproductive outcomes?

To address these questions, this analysis uses a linear mixed-effects model (Bates et al. (2014)) with fixed effects for female population density, spatial location, and age, and a random intercept for year. Moreover, it incorporates an AR(1) (Box et al. (1994)) correlation structure to account for temporal autocorrelation. The results show that population and nesting location significantly affect reproductive success, while age has no statistically significant impact. Temporal variability is effectively captured by the random intercept and the AR(1) structure, highlighting year-to-year differences in reproductive outcomes. This study highlights the need to account for both fixed and random effects to understand ecological processes and suggests future model improvements, such as exploring non-linear patterns and additional ecological variables.

2 STUDY DESIGN

The study followed a longitudinal design with repeated measurements for some individuals over multiple years.

Data collection: During the 19-year study, all observed song sparrows were tagged and assigned a unique identifier. For 15 of these years, researchers conducted regular visits to each nest every 2–7 days throughout the mating season. During these visits, newly hatched offspring ("fledglings") were tagged to track individual reproductive success over time. The total number of offspring, which is the primary measure of reproductive success, was then recorded for a particular sparrow in a specific year.

Data description: The dataset consists of 742 entries corresponding to observations of 360 unique song sparrows over a 19-year period. Since many sparrows were observed repeatedly across multiple years, a single sparrow may have multiple entries in the dataset. Each entry represents a specific sparrow observed during a particular year. The key columns in the dataset are as follows:

- `band`: A unique identifier assigned to each sparrow.
- `cohort`: The year when the sparrow was first tagged (ranging from 2 to 19).
- `year`: The year of observation (ranging from 1 to 19).
- `flop`: The total number of female sparrows in the population during the given year.
- `age`: The age of the sparrow at the time of observation.
- `x,y`: The spatial coordinates of the sparrow's nest on the island.
- `spf` (Target Variable): The number of offspring (fledglings) produced by the sparrow in that year.

In this analysis, we chose to remove rows with missing values (`data_clean`) to maintain consistency across variables and to facilitate straightforward interpretation of the final model results. After this step, the dataset consists of 641 entries and 8 columns. While imputation methods could be considered, their implementation might introduce additional assumptions that are beyond the scope of this study. By excluding missing rows, we ensure the model is fit only to complete cases, which allows for robust parameter estimation and valid inference.

Macro Explanatory Variables: The macro variables in this dataset are those that capture factors affecting the reproductive success of sparrows at a broader, population-wide, or environmental level. Unlike micro variables (which are specific to each individual bird), macro variables influence all individuals in the population during a specific time period.

- `year`: captures changes in environmental conditions, resource availability, and other temporal effects that may influence reproductive success. Different years may reflect variations in weather, food availability, or predator activity, all of which can impact the number of offspring produced by individual birds.
- `fpop` (Population Density): reflects competition for limited resources such as food, mates, and nesting sites. As population density increases, competition intensifies, which could reduce reproductive success. This variable captures population-wide effects on individual outcomes.
- `cohort`: captures generation-level effects, as sparrows tagged in different years might face distinct environmental conditions or selective pressures. Sparrows from earlier cohorts may have had different survival or reproductive conditions than those from later cohorts.

Micro Explanatory Variables: Micro variables capture characteristics that are unique to each bird, influencing its own reproductive success.

- `band`: allows for tracking individual birds across multiple years. It enables the inclusion of individual-specific random effects in statistical models, allowing for variation in reproductive success across birds due to unobserved individual traits like genetic fitness or learned behaviors.
- `age`: affects reproductive success, as younger or older birds may produce fewer offspring compared to birds at their peak reproductive age. This variable allows for analysis of life-stage effects on reproduction.
- `x, y` (Nest Coordinates): The location of a bird's nest may influence its reproductive success. Birds nesting in more favorable locations (e.g., areas with higher food availability or protection from predators) may produce more offspring. This variable allows for spatial analysis of reproductive success.
- `cohort`: bridges the gap between macro and micro explanatory variables. As a macro variable, cohort reflects the environmental and ecological conditions present during the year a sparrow was first tagged, influencing the survival and reproductive opportunities of all sparrows tagged that year. As a micro variable, it is specific to each sparrow and, combined with year, determines the bird's age (`age = year - cohort + 1`), which directly impacts its reproductive success.

3 Data Description

In this section, we will examine the relationships between the number of offspring and grouping factors such as year, cohort, and band. Additionally, we will analyze the interplay between cohort, age, and year, as well as address any missing data issues in the dataset.

3.1 Visualizing the Relationship Between the Number of Offspring (`spf`) and Grouping Factors:

We first investigate the number of observations across two grouping factors. Sample sizes fluctuate significantly by year and cohort, with some years having notably fewer observations. This variability highlights the importance of accounting for unbalanced sample sizes during data analysis.

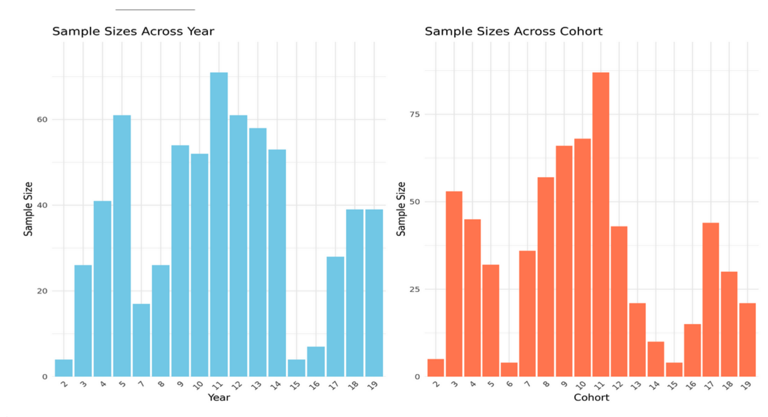

Figure 1: Sample Sizes Across Year and Cohort

Next, the variability in offspring numbers (`spf`) was examined using box plots grouped by `year`, `cohort`, and `band` (individual birds).

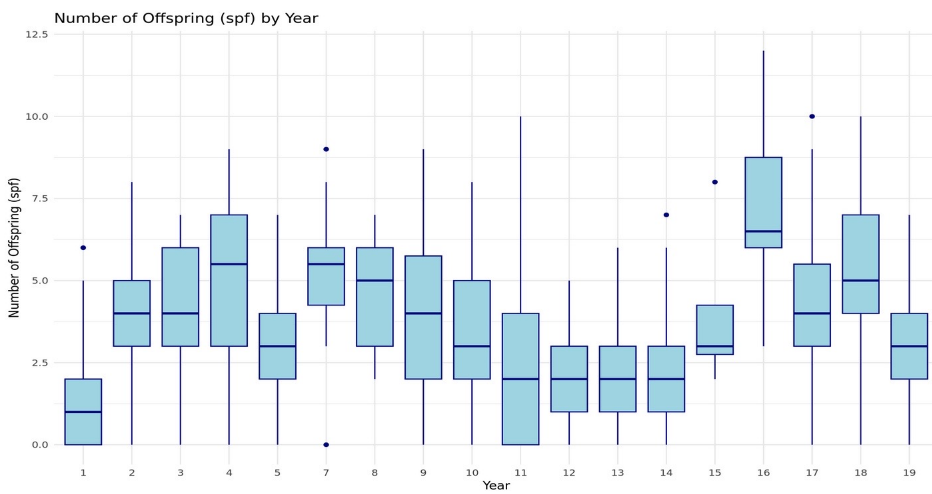

Figure 2: Box plot of Number of Offspring by Year

The box plot for `spf` by Year (Figure 1) shows noticeable variability in the number of offspring produced across different years. Some years exhibit a higher median number of offspring (e.g., Year 4, 7, 16), while others show lower offspring counts overall (e.g., Year 1 and Years 11–14). Outliers are also present, indicating that in some years, certain birds produced significantly more offspring than others. This suggests that yearly environmental conditions, such as weather, food availability, or predator pressure, likely play a role in influencing reproductive success. Including `year` in the model as a factor might help account for this temporal variability.

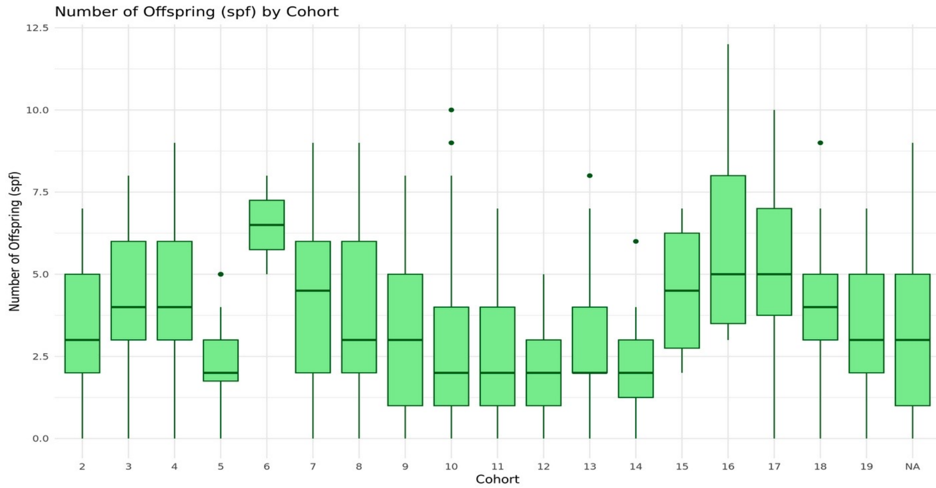

Figure 3: Box plot of Number of Offspring by Cohort

The box plot for `spf` by Cohort (Figure 2) reveals variability in reproductive success depending on the year a bird was first tagged (its cohort). Birds from certain cohorts (e.g., Cohort 6) show higher median offspring counts, while others (e.g., Cohorts 10–12) show lower counts. These differences may reflect ecological or environmental conditions present during a bird's birth year, which might have long-term effects on its reproductive success. We will then investigate the relationship between `year`, `cohort`, and `age` to further understand these patterns.

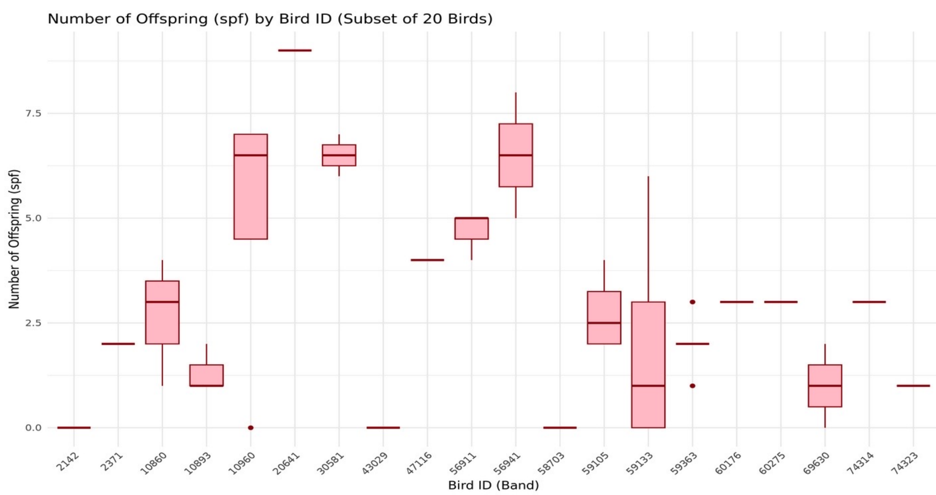

Figure 4: Box plot of Number of Offspring by Band (20 Bands Randomly Selected)

The box plot for `spf` by Band (Figure 3) highlights clear differences in reproductive success among individual birds, even within the same year. Some birds consistently produce more offspring than others, potentially due to genetic fitness, learned behaviors, or differences in access to resources. Bird-specific factors, such as genetics, experience, or territory quality, contribute to this variability. Including Bird ID (Band) as a random effect in a hierarchical model might help account for this individual-level variability.

3.2 Relationship Between Cohort, Year, and Age

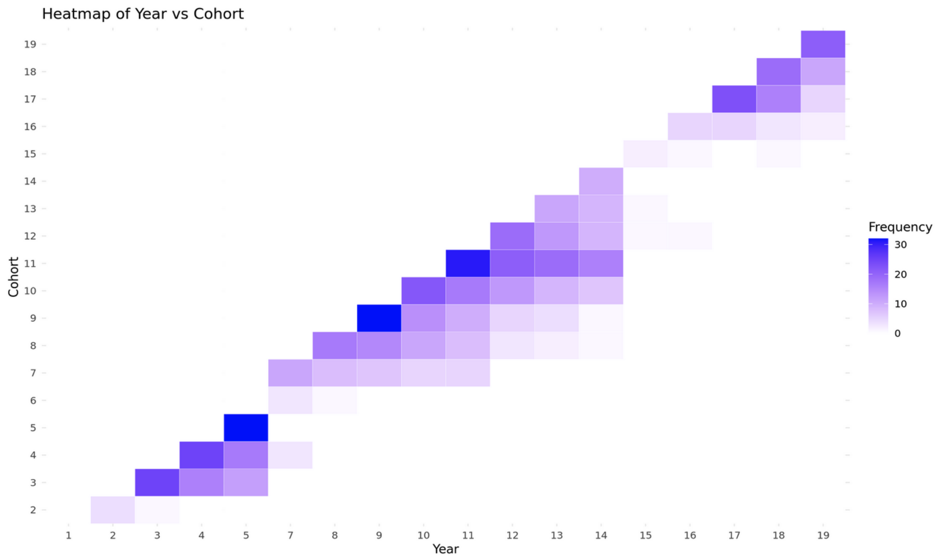

Figure 5: Heat map of Year vs Cohort (Frequency)

The heatmap above summarizes the cross-tabulation between year and cohort. The majority of non-zero counts are concentrated along the diagonal, indicating that year and cohort are closely aligned. This alignment occurs because most sparrows tagged in a specific cohort were first observed in the corresponding year. However, some off-diagonal entries represent instances where birds from earlier cohorts survived and were observed in subsequent years.

The relationship among year, cohort, and age can be described by the formula:

```
age = year - cohort + 1
```

This formula highlights the linear dependency between these variables, which is further supported by the following correlations:

- The correlation between year and cohort is 0.968, indicating a very strong linear relationship. This strong alignment suggests potential multicollinearity, as including both variables as fixed effects in the same model could inflate standard errors and hinder the isolation of their individual contributions to reproductive success.
- The correlation between year and age is 0.168, suggesting a weak positive association.
- The correlation between cohort and age is -0.076, indicating a very weak negative relationship.

These results confirm that while year and cohort are nearly collinear, age provides additional, distinct information about the life stage of individual birds. Careful consideration of these relationships is crucial to avoid overfitting or misinterpretation in the modeling process.

3.3 Preliminary Modeling

To gain preliminary impressions about the effects of age, female population, and nesting location on the number of offspring, we consider the following points:

- To address the main research question, biologically relevant variables such as age, competition from other females (female population), and nesting locations will be included in the preliminary model.
- Based on boxplot visualizations, the number of offspring appears to vary across year, cohort, and band (individual sparrows). To account for this variability, these variables will be considered in the preliminary model.
- Given the high correlation between year and cohort (correlation coefficient: 0.968), including both variables as fixed effects may lead to multicollinearity. We will perform hypothesis testing to evaluate their contributions and decide how best to include them in future models.

- When building the model, we will carefully check the following assumptions:
  - Linearity: Ensure a linear relationship between predictors and the response variable.
  - Constant Variance: Verify homoscedasticity (constant variance of residuals).
  - Independence: Be aware that independence may be violated due to repeated measures for individual sparrows.

3.3.1 Sequential Hypothesis Testing

Micro Variables

To evaluate the contributions of micro-level explanatory variables to reproductive success (`spf`), we performed a Type III ANOVA on a linear model including all relevant predictors: `age`, `fpop`, and the spatial coordinates (`x`, `y`):

$$\texttt{lm\_full:}\quad \texttt{spf} \sim \texttt{fpop} + \texttt{age} + \texttt{x} + \texttt{y}$$

The results indicate that:

- Female population `fpop` is the strongest predictor ($F = 88.66, p < 0.001$), emphasizing the critical role of competition for resources in determining reproductive success.
- The `x`-coordinate shows a significant spatial effect ($F = 10.47, p = 0.001$), suggesting that certain nesting locations may provide favorable conditions.
- The variable `age` is marginally significant ($F = 3.09, p = 0.079$), reflecting a potential, though weaker, effect of individual maturity.
- In contrast, the `y`-coordinate does not contribute significantly ($F = 0.02, p < 0.1$), indicating no evidence for an independent spatial effect in that direction.

Moving forward, we will focus on these variables while simplifying the model by excluding `y` due to its lack of significance. The linear model is defined as:

$$\texttt{lm:}\quad \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age}$$

Macro Variables

To explore the contributions of temporal variability and cohort effects on reproductive success, we conducted a series of linear model analyses comparing different combinations of predictors. To assess the additional effects of `year` and `cohort`, we systematically compared nested models using `ANOVA` and information criteria (AIC and BIC (Chakrabarti & Ghosh (2011))). The models under consideration are:

$$\begin{aligned}
\texttt{lm} &: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} \\
\texttt{lm_inter1} &: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{fpop:age} \\
\texttt{lm_inter2} &: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{x:y} \\
\texttt{lm_year} &: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{as.factor(year)} \\
\texttt{lm_cohort} &: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{as.factor(cohort)} \\
\texttt{lm_both} &: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{as.factor(year)} + \texttt{as.factor(cohort)}
\end{aligned}$$

Regarding the baseline models, both the BIC and AIC of the lm model without the interaction terms(either `fpop:age` or `x:y`) is the lowest, suggesting that adding interactions does not improve model fit sufficiently to justify the increased complexity.

Next we want to test the effect of adding either `year` or `cohort` to the chosen baseline model. We notice that:

- Adding `year` to the baseline model significantly improved the fit using `anova(lm, lm_year)`. The small p-value ($F = 6.48, p < 0.001$) confirmed that `year` has a significant effect on the number of offspring (`spf`) beyond the contributions of `age` and `fpop`. This result suggests that temporal variability, possibly driven by yearly environmental factors such as weather, food availability, or predation pressure, plays an important role in reproductive success.

- Using anova(lm, lm_cohort), we see that cohort also significantly improves the model fit ($F = 4.48, p < 0.001$), likely reflecting long-term ecological or generational effects on reproductive success.

Next we want to test the effect of adding either year or cohort while controlling for the other variable. We come up with the following observations:

- To test whether adding year to a model that already includes cohort significantly improves the fit, we used anova(lm_cohort, lm_both). The p-value ($F = 3.37, p < 0.001$) confirmed that year provides additional explanatory power for spf after controlling for cohort.
- We tested whether adding cohort to a model that already includes year using anova(lm_year, lm_both). The results also showed a significant improvement ($F = 3.37, p < 0.001$), but the AIC for the combined model (lm_both) was slightly higher than that of the model with year alone (lm_year). This suggests that the added complexity of including both variables may not be justified. Furthermore, the high correlation ($r = 0.968$) between year and cohort introduces redundancy and multicollinearity, making it challenging to separate their effects.

Now, we compare the AIC and BIC for all the mentioned models:

- The AIC for the model with year (lm_year) was the lowest, indicating the best balance between model fit and complexity for prediction.
- For both AIC and BIC, the model with cohort (lm_cohort) performed worse than the model with year, suggesting that year explains more variability in the data.
- Although the BIC slightly favored the simpler baseline model (lm), the strong significance of year in the ANOVA results justifies its inclusion.

Based on these results:

- anova(lm, lm_year) tests the effect of year on spf and shows strong evidence for its inclusion.
- anova(lm, lm_cohort) tests the effect of cohort on spf and shows weaker but significant evidence for its inclusion.
- anova(lm_cohort, lm_both) tests the effect of year on spf after controlling for cohort, showing significant improvement.
- anova(lm_year, lm_both) tests the effect of cohort on spf after controlling for year, indicating some additional value but limited improvement.

In conclusion, we selected the model with year (lm_year) as the most appropriate for capturing temporal variability. This model provides the best balance between explanatory power and complexity while minimizing redundancy. Moving forward, we will validate the assumptions of this model before proceeding to further analyses.

3.3.2 Checking Assumptions

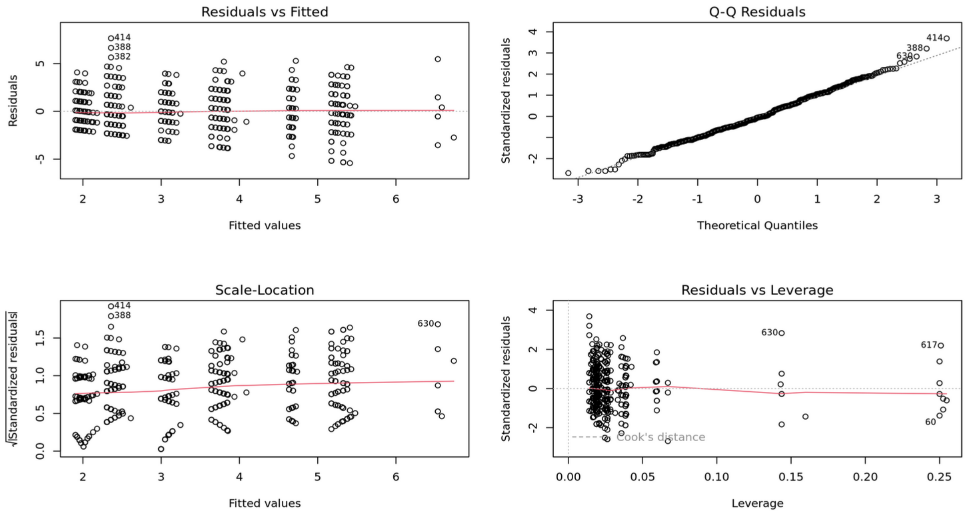

Figure 6: Diagnostic plots for assumptions (`lm_year`)

Based on the diagnostic plots provided, we consider the following points:

Residuals vs Fitted: The plot shows that the residuals are scattered randomly around the horizontal line at zero, suggesting that the assumption of linearity is reasonably satisfied. However, some points deviate slightly, indicating possible heteroscedasticity or non-linearity for certain fitted values.

Q-Q Plot: The residuals mostly follow the diagonal line, indicating that the normality assumption of residuals is generally satisfied. However, there are slight deviations in the tails, suggesting potential outliers or mild deviations from normality.

Scale-Location: The spread of the standardized residuals across fitted values appears relatively consistent, supporting the assumption of homoscedasticity. Some points show slight variations in spread, but no clear patterns indicate a severe violation of this assumption.

Residuals vs Leverage: The plot shows that most residuals are within acceptable ranges of leverage, with a few potential high-leverage points (e.g., Cook's distance lines indicating influential points). These points may warrant further investigation, as they could disproportionately affect the model.

In summary, the assumptions of linearity, normality, and homoscedasticity are generally met with minor deviations.

4 Model Fitting and Diagnostics

4.1 Linear Mixed-Effects Models

To fully capture the variability in reproductive success (`spf`), we expanded upon the preliminary linear model (`lm_year`) by developing a hierarchical (mixed-effects) model Bates et al. (2014). This approach accounts for the structured nature of the dataset, where observations are nested within grouping factors such as **year** (`year`) and **individual birds** (`band`), potentially introducing correlations between observations.

The hierarchical model includes:

- Fixed Effects:
  - fpop: Female population density, to account for competition effects.
  - x: Spatial coordinate, to account for location-based variability.
  - age: Bird age, as a proxy for reproductive maturity.
- Random Effects:
  - Band: Individual birds, to account for repeated measures and variability between individuals.

– Year: Temporal variability, accounting for year-specific effects.

We now consider the following models:

$$\texttt{fit1}: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{(1 | band)} + \texttt{(1 | year)}$$
$$\texttt{fit2}: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{(1 | year)}$$
$$\texttt{fit3}: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age} + \texttt{(1 | band)}$$

The model with the lowest BIC, `fit2`, includes a random intercept for `year` only. This is also the model with the lowest BIC out of all the models we have considered so far. This suggests that temporal variability across years is the most critical source of random variation in the dataset.

The model (`fit2`) can be expressed mathematically as:

$$\texttt{spf}_{ij} = \beta_0 + \beta_1 \cdot \texttt{fpop}_{ij} + \beta_2 \cdot \texttt{x}_{ij} + \beta_3 \cdot \texttt{age}_{ij} + u_j + \epsilon_{ij}$$

Where:

- $\texttt{spf}_{ij}$: Reproductive success for the i-th observation in the j-th year.
- β_0: Global intercept.
- $\beta_1, \beta_2, \beta_3$: Fixed-effect coefficients for `fpop`, `x`, and `age`.
- $u_j \sim \mathcal{N}(0, \sigma_u^2)$: Random intercept for `year`, capturing year-specific variability.
- $\epsilon_{ij} \sim \mathcal{N}(0, \sigma_\epsilon^2)$: Residual error term for the i-th observation in the j-th year.

This model captures the fixed effects of the explanatory variables (`fpop`, `x`, `age`) and accounts for year-specific variability using a random intercept (u_j).

4.2 Modeling Temporal Correlation

To account for temporal autocorrelation in the dataset, we explored hierarchical models that incorporate an **autoregressive structure** of order 1 (AR(1)) (Box et al. (1994)) for the residuals. Temporal autocorrelation is a natural concern in this dataset since observations are grouped by year, and environmental factors influencing reproductive success may persist across consecutive years.

Our modeling approach is a linear mixed-effects model was fit with random intercepts for `year` and an AR(1) correlation structure. This model assumes that observations from consecutive years are more correlated than those from non-consecutive years, and it explicitly models this dependence. Additionally, we explored a model with both random intercepts and slopes for `age` by `year`, incorporating an AR(1) structure. This allows the effect of age on reproductive success to vary across years, capturing potential year-specific variability in age-related reproductive patterns.

$$\texttt{fit4}: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age}, \texttt{random=1|year}, \texttt{correlation = corAR1()}$$
$$\texttt{fit5}: \texttt{spf} \sim \texttt{fpop} + \texttt{x} + \texttt{age}, \texttt{random=age|year}, \texttt{correlation = corAR1()}$$

The model with AR(1) structure and only random intercepts for year (`fit4`) was selected over (`fit5`), as it strikes the best balance between model fit and complexity. It has the lowest BIC among all the models under consideration in this analysis. The slightly higher BIC for `fit5` suggests that adding random slopes for age by year does not sufficiently improve the model to justify the added complexity.

To explore possible **interactions** between micro variables in `fit4`, such as `fpop*age,fpop*x, x*y, fpop*y, age*y, etc`, we used the `add1()` function. This function evaluates whether including a specific interaction term significantly improves the model fit. The addition of an interaction term was tested using a likelihood ratio test (LRT), with a null hypothesis (H_0) that the interaction term does not significantly improve the model. Based on the LRT results, none of the interaction terms considered were statistically significant.

The final selected model (`fit4`) can be expressed mathematically as:

$$\texttt{spf}_{ij} = \beta_0 + \beta_1 \cdot \texttt{fpop}_{ij} + \beta_2 \cdot \texttt{x}_{ij} + \beta_3 \cdot \texttt{age}_{ij} + u_j + \epsilon_{ij}$$

Where:

- $\texttt{spf}_{ij}$: Reproductive success for the i-th observation in the j-th year.
- β_0: Global intercept.
- $\beta_1, \beta_2, \beta_3$: Fixed-effect coefficients for `fpop`, `x`, and `age`.
- $u_j \sim \mathcal{N}(0, \sigma_u^2)$: Random intercept for `year`, accounting for year-specific variability.
- $\epsilon_{ij} \sim \mathcal{N}(0, \sigma_\epsilon^2)$: Residual error term with AR(1) structure: $\epsilon_{ij} = \phi \cdot \epsilon_{ij-1} + \eta_{ij}$, where ϕ is the autoregressive parameter and $\eta_{ij} \sim \mathcal{N}(0, \sigma_\eta^2)$.

The AR(1) correlation structure in the residuals explicitly accounts for the temporal dependence between observations from consecutive years.

4.3 CHECKING ASSUMPTIONS

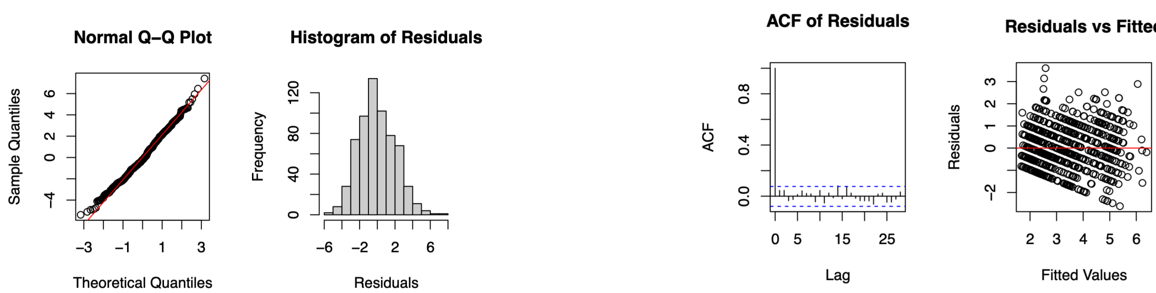

Figure 7: Diagnostic plots for assumptions (fit4)

The normal Q-Q plot and the histogram of residuals show that most residuals lie close to the diagonal line, indicating that the residuals are approximately normally distributed. However, there are some deviations at the tails, suggesting mild violations of normality, particularly in extreme residuals.

The ACF plot shows no significant autocorrelation in the residuals, as all lag values lie within the confidence bounds. This result supports the AR(1) correlation structure used in the model and suggests that temporal autocorrelation has been adequately accounted for.

The residuals vs. fitted values plot shows no clear pattern, indicating that the assumption of homoscedasticity (constant variance of residuals) holds. The residuals are randomly scattered around zero, suggesting that the linearity assumption is also satisfied.

5 MODEL AND DATA ANALYSIS INTERPRETATION

We conducted a comprehensive summary of model `fit4` using the functions `summary()` and `intervals()`, yielding the following results and interpretations.

Fixed Effects	Value	Lower 95% CI	Upper 95% CI	Std. Error	DF	t-value	p-value
(Intercept)	6.7003	5.4502	7.9504	0.6386	617	10.4922	0.0000
`fpop`	-0.0542	-0.0809	-0.0275	0.0126	15	-4.3165	0.0006
`x`	-0.0337	-0.0538	-0.0136	0.0103	617	-3.2855	0.0011
`age`	0.0491	-0.1195	0.2176	0.0861	617	0.5699	0.5690

Table 1: Summary of fixed effects for the model `fit4`.

Fixed Effects:

- **Intercept:** The estimated intercept is 6.70, representing the expected number of offspring (`spf`) when all other explanatory variables (`fpop`, `x`, `age`) are at their baseline levels. The intercept is statistically significant ($p < 0.0001$).

The 95% confidence interval for the intercept is [5.45, 7.95], indicating that the expected number of offspring (`spf`) at baseline levels of `fpop`, `x`, and `age` lies within this range. The narrow confidence interval reflects precise estimation.

- **Female Population (`fpop`):** The coefficient is -0.054, indicating that for every unit increase in female population density, the number of offspring decreases by 0.054, holding other variables constant. This effect is statistically significant ($p = 0.0006$).

  The 95% confidence interval for the effect of female population density is [-0.081, -0.028]. This confirms that increasing `fpop` has a statistically significant negative impact on `spf`, as the interval does not include zero.

- **Spatial Variable (`x`):** The coefficient is -0.034, showing that for every unit increase in the spatial variable, the number of offspring decreases by 0.034, holding other variables constant. This effect is statistically significant ($p = 0.0011$).

  The confidence interval for the effect of `x` is [-0.053, -0.014], reinforcing the conclusion that `x` has a statistically significant negative association with `spf`.

- **Age:** The coefficient is 0.049, suggesting a small positive relationship between bird age and the number of offspring. However, this effect is not statistically significant ($p = 0.569$).

  The 95% confidence interval for the effect of `age` is [-0.119, 0.218]. Since the interval includes zero, the effect of `age` is not statistically significant.

Description	Value	Lower 95% CI	Upper 95% CI
Random Effects			
`sd((Intercept))` (Level: year)	0.7567	0.4741	1.2080
`Residual StdDev`	2.0592	1.9459	2.1792
Correlation Structure (AR(1))			
Φ (Phi)	0.0913	0.0086	0.1727
Within-group Standard Error			
Standard Error	2.0592	1.9459	2.1792

Table 2: Summary of random effects, correlation structure, and within-group standard error.

Random Effects:

- **Year:** The standard deviation of the random intercept for `year` is 0.757, with 95% confidence interval being [0.474, 1.208], capturing the variability in `spf` across years. This indicates that there are differences in reproductive success across years that are not fully explained by the fixed effects.
- **Residuals:** The residual standard deviation is 2.059, representing the unexplained variability within groups (years).

Correlation Structure (AR(1)): The estimated autoregressive parameter (ϕ) is 0.091, indicating a weak positive correlation between observations in consecutive years within the same group. The confidence interval for ϕ [0.009, 0.173] supports the presence of weak but statistically significant temporal autocorrelation.

Within-Group Residuals: The within-group standard error is estimated at 2.059, with a 95% confidence interval of [1.946, 2.179]. This reflects the variability within groups that remains unexplained after accounting for both fixed and random effects.

Summary: Based on the detailed results above, we draw the following high-level conclusions:

- Female population density (`fpop`) and the spatial variable (`x`) are significant predictors of reproductive success (`spf`), whereas age does not exhibit a statistically significant effect.
- The random intercept for `year` and the AR(1) correlation structure effectively capture temporal variability in the data, indicating that observations are correlated over time.
- While the fixed effects explain a portion of the variability in `spf`, some variability remains unexplained, as reflected in the residual standard deviation.

6 CONCLUSIONS

Our analysis of the reproductive success (spf) of sparrows reveals several key scientific findings:

Summary of Hypothesis Tests and Model Selection:

- Hypothesis tests for micro-level variables revealed that female population density (fpop) and the spatial variable (x) were significant predictors of reproductive success, while age did not show statistical significance.
- Comparisons of models using AIC and BIC demonstrated that including year as a random intercept provided a better balance between model complexity and predictive power compared to models with cohort or both year and cohort.
- The selected model (fit4) incorporates fixed effects for fpop, x, and age, a random intercept for year, and an AR(1) correlation structure for residuals. This model effectively captures temporal variability and explains a significant portion of the observed variance in spf.

Parameter Estimates:

- The intercept (6.70) represents the expected number of offspring (spf) when all other explanatory variables are at their baseline levels.
- The coefficient for fpop (-0.054) indicates a significant negative effect of female population density on reproductive success.
- The coefficient for x (-0.034) shows a significant spatial effect, suggesting that certain locations reduce the number of offspring.
- The coefficient for age (0.049) was positive but not statistically significant, implying a weak or negligible effect of age on reproductive success.
- The random intercept for year has a standard deviation of 0.757, capturing the unexplained variability in reproductive success across years.
- The AR(1) correlation parameter ($\phi = 0.091$) indicates a weak but statistically significant positive correlation between consecutive years within the same group, with a confidence interval of $[0.009, 0.173]$.
- The within-group standard deviation is 2.059, with a confidence interval of $[1.946, 2.179]$, reflecting variability not explained by the fixed and random effects.

Scientific Conclusions:

- Female population density (fpop) and spatial variability (x) are key factors influencing reproductive success, while temporal variability is captured effectively through the random intercept for year and the AR(1) correlation structure.
- Age does not significantly impact reproductive success in this dataset, suggesting its influence may be minor compared to other predictors.
- The fixed effects explain a substantial portion of the variability in spf, although residual variability remains.

Limitations:

- The analysis assumes linearity and normality of residuals, which may not fully capture complex relationships or interactions among predictors.
- Correlations between year and cohort introduce multicollinearity, making it difficult to separate their effects.
- Spatial heterogeneity may not be fully captured due to the linear treatment of x and y.
- The AR(1) structure assumes that the correlation between consecutive years is constant, which may not fully reflect the complexity of temporal patterns in the data.

Future Studies:

- Incorporate additional ecological variables, such as food availability and predator density, to provide a more comprehensive understanding of factors affecting reproductive success.

- Explore non-linear modeling techniques, such as generalized additive models (GAMs) or machine learning, to capture complex patterns in the data.
- Conduct longitudinal studies with more detailed temporal data to refine our understanding of temporal variability and year-to-year changes.

These conclusions emphasize the importance of understanding both fixed and random effects in explaining reproductive success, while identifying potential avenues for further investigation and model refinement.

References

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A Appendix

Model Name	**Model Specification**	**BIC**
`lm`	`spf` ∼ `fpop` + `x` + `age`	2826.518
`lm_inter1`	`spf` ∼ `fpop` + `x` + `age` + `fpop:age`	2832.744
`lm_inter2`	`spf` ∼ `fpop` + `x` + `age` + `x:y`	2832.907
`lm_cohort`	`spf` ∼ `fpop` + `x` + `age` + `as.factor(cohort)`	2862.001
`lm_year`	`spf` ∼ `fpop` + `x` + `age` + `as.factor(year)`	2830.216
`lm_both`	`spf` ∼ `fpop` + `x` + `age` + `as.factor(year)` + `as.factor(cohort)`	2908.993
`fit1`	`spf` ∼ `fpop` + `x` + `age` + `(1 | band)` + `(1 | year)`	2809.272
`fit2`	`spf` ∼ `fpop` + `x` + `age` + `(1 | year)`	2806.269
`fit3`	`spf` ∼ `fpop` + `x` + `age` + `(1 | band)`	2843.636
`fit4`	`spf` ∼ `fpop` + `x` + `age` + `(1 | year)` with AR(1) structure	**2789.482**
`fit5`	`spf` ∼ `fpop` + `x` + `age` + `(age | year)` with AR(1) structure	2791.575

Table A3: Comparison of models based on their specifications and BIC values.